Math 005 Lecture Notes
10-30-2009

Section 5.3 - Graphing Sine and Cosine

1 Things to Note About Sine and Cosine

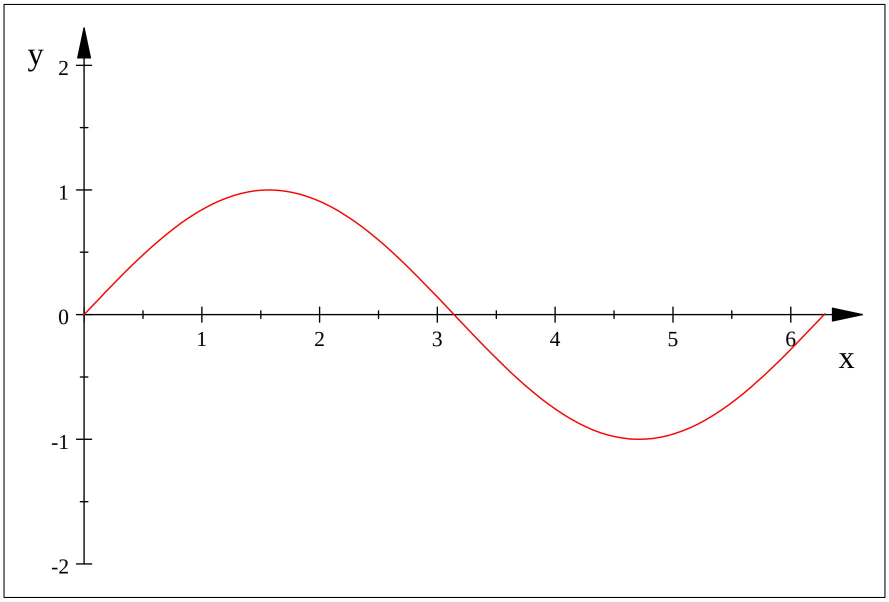
$\sin(-x) = -\sin(x)$ (sine is odd)
$\cos(-x) = \cos(x)$ (cosine is even)

2 Graphing $y = sin(x)$

Consider the following chart of values:

$x =$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin x =$	0	1	0	-1	0

If we plot these points we get the following graph:



Sine is what we call a periodic function.

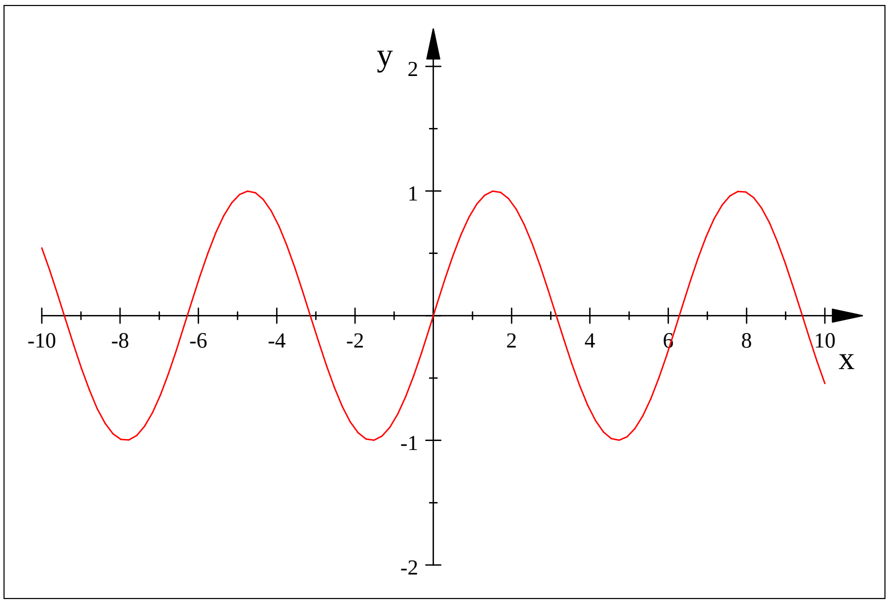
Definition 1 *A* periodic function *is a function $y = f(x)$ where $f(x) = f(x + a)$ for some number a. The smallest such positive a is called the* period *of f.*

Definition 2 *The* amplitude *of a sine wave is the absolute value of half the difference between the maximum and minimum y-values of the wave:*

$$\textit{Amplitude} = A = \frac{1}{2}\left|y_{\max} - y_{\min}\right|.$$

Using the fact that $y = \sin x$ is periodic, we can extend it to the whole real line and the graph looks like:

$y = \sin x$

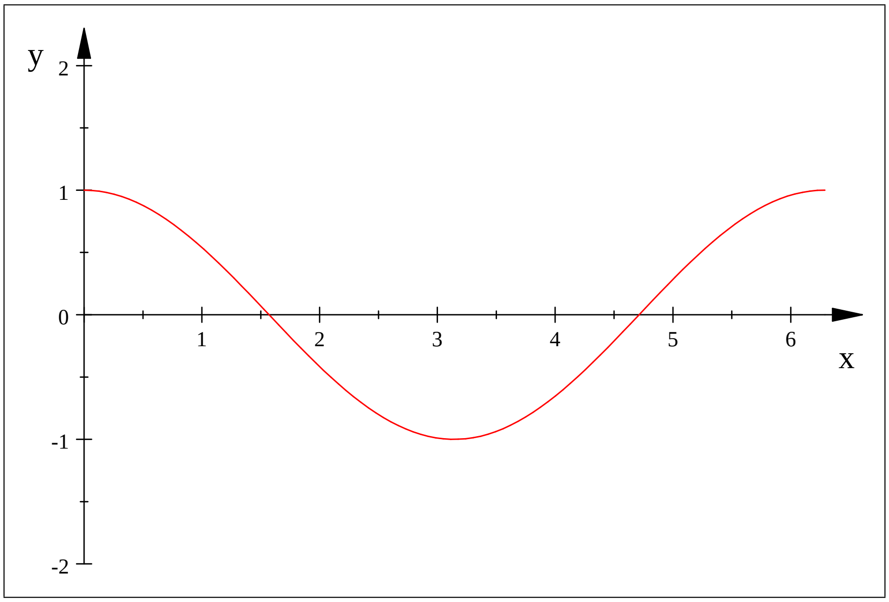


3 Graphing $y = \cos x$

Consider the following chart of values:

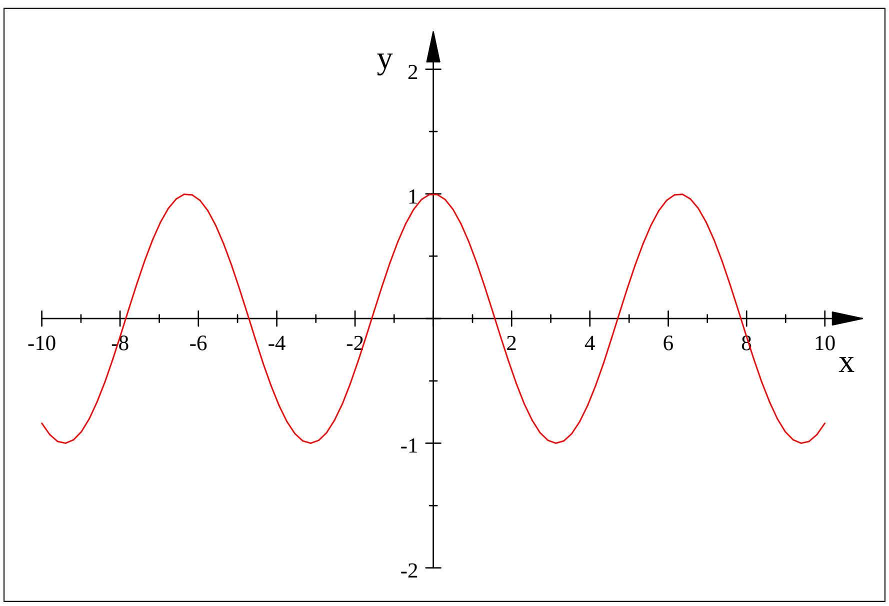
$x =$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\cos x =$	1	0	-1	0	1

If we plot these points we get the following graph:



Cosine is also a periodic function. It is also a sine wave.

Using the fact that $y = \cos x$ is periodic, we can extend it to the whole real line and the graph looks like:



4 Properties of The Graphs

Definition 3 *The* amplitude *of* $y = A\sin x$ *or* $y = A\cos x$ *is* $|A|$.

Definition 4 *The* phase shift *of* $y = \sin(x - C)$ *or* $y = \cos(x - C)$ *is* C.

Definition 5 *The* period *of* $y = \sin(Bx)$ *or* $y = \cos(Bx)$ *is* $P = \frac{2\pi}{B}$.

Remark 6 *To graph* $y = \sin(Bx)$ *you can use the following chart:*

$x =$	0	$\frac{\frac{\pi}{2}}{B}$	$\frac{\pi}{B}$	$\frac{\frac{3\pi}{2}}{B}$	$\frac{2\pi}{B}$
$\sin(Bx) =$	0	1	0	-1	0

Similarly for $y = \cos(Bx)$ *you can use this chart:*

$x =$	0	$\frac{\frac{\pi}{2}}{B}$	$\frac{\pi}{B}$	$\frac{\frac{3\pi}{2}}{B}$	$\frac{2\pi}{B}$
$\cos(Bx) =$	1	0	-1	0	1

5 Families of Sine and Cosine

Definition 7 *The families of sine and cosine look like*

$$y = A\sin[B(x - C)] + D \text{ or } y = A\cos[B(x - C)] + D,$$

and have amplitude $|A|$, *period* $\frac{2\pi}{B}$ $(B > 0)$, *phase shift (horizontal translation)* C, *and vertical translation* D.

Example 8 *Sketch 2 cycles (periods) of* $y = 2\sin(3x + \pi) + 1$.

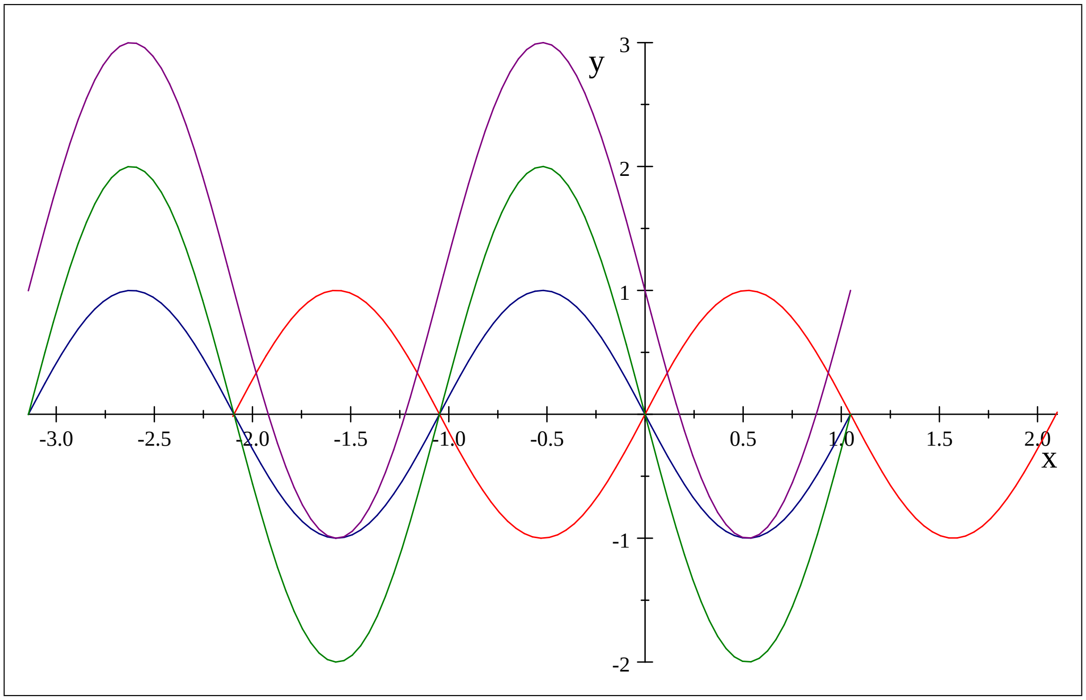
Solution:

Steps for solving the problem:

1. *Put* $y = 2\sin(3x + \pi)$ *in standard form*

$$y = 2\sin\left[3\left(x + \frac{\pi}{3}\right)\right]$$

2. *Graph* $y = \sin 3x$. *It has period* $\frac{2\pi}{3}$. *(This is graphed in red.)*
3. *Translate the graph to the left by* $\frac{\pi}{3}$. *(This is graphed in blue.)*
4. *Change the amplitude to* 2. *(This is graphed in green.)*
5. *Translate* 1 *unit upward. (This is graphed in purple.)*



Example 9 *Sketch 2 cycles (periods) of* $y = -3\cos(2x - \pi) - 1$.

Solution:

Steps for solving the problem:

1. *Put* $y = -3\cos(2x - \pi) - 1$ *in standard form*

$$y = -3\cos\left[2\left(x - \frac{\pi}{2}\right)\right] - 1$$

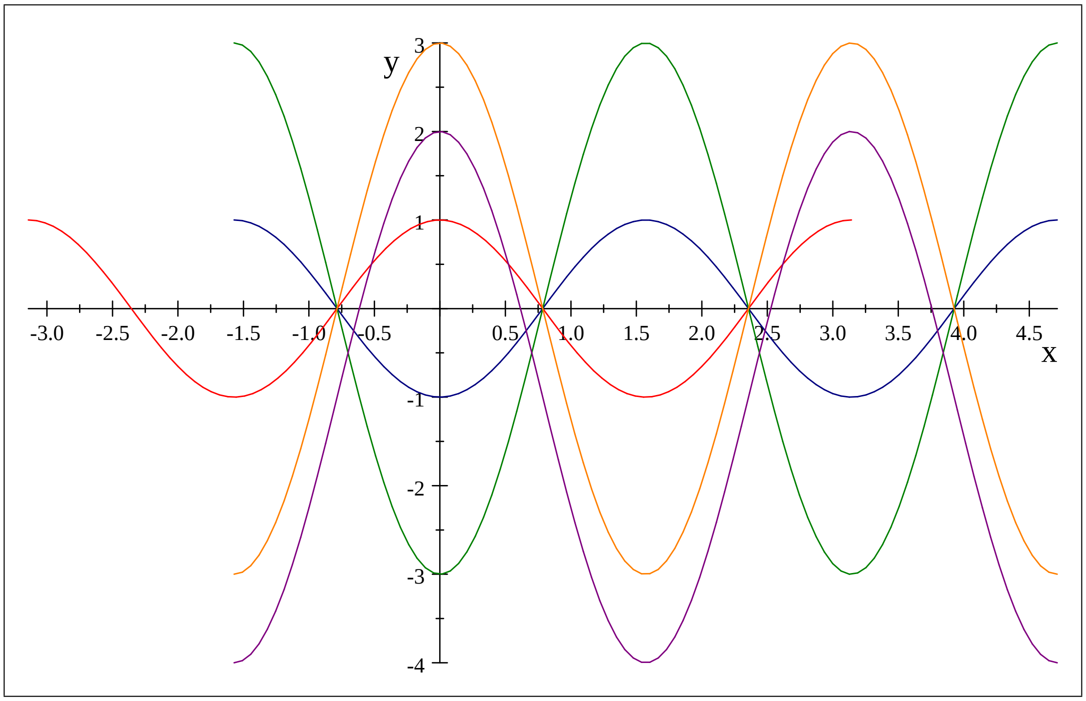
2. *Graph* $y = \cos 2x$. *It has period* π. *(This is graphed in red.)*

3. *Translate the graph to the right by* $\frac{\pi}{2}$. *(This is graphed in blue.)*

4. *Change the amplitude to* 3. *(This is graphed in green.)*

5. *Since* $A < 0$, *flip the graph over the* $x-axis$. *(This is graphed in orange.)*

6. *Translate* 1 *unit downward. (This is graphed in purple.)*

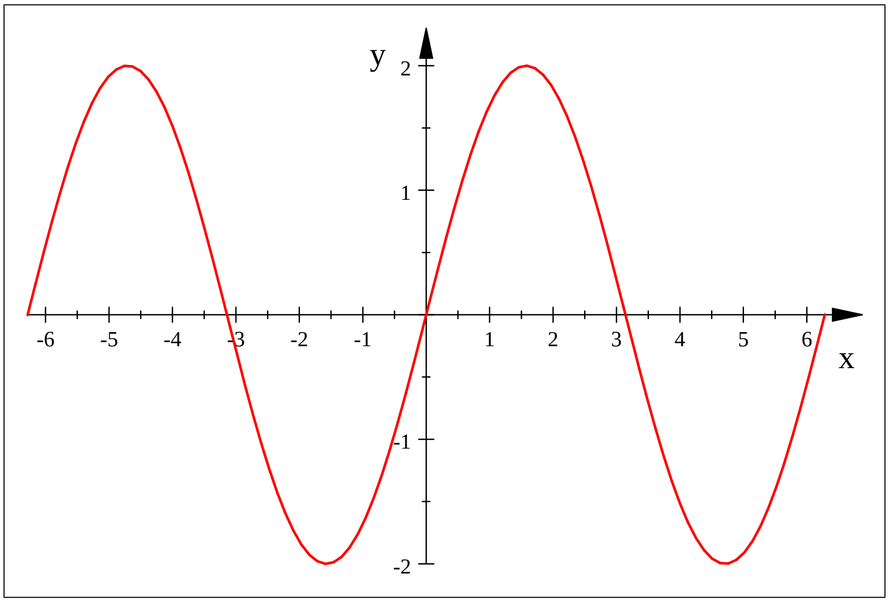


Problem 10 *This was going to be the problem of the day:*

Graph 2 cycles of :

$$y = 2\sin(\pi - x)$$

The graph looks like:


y
2
1
-1
-2
-6
-5
-4
-3
-2
-1
1
2
3
4
5
6
x